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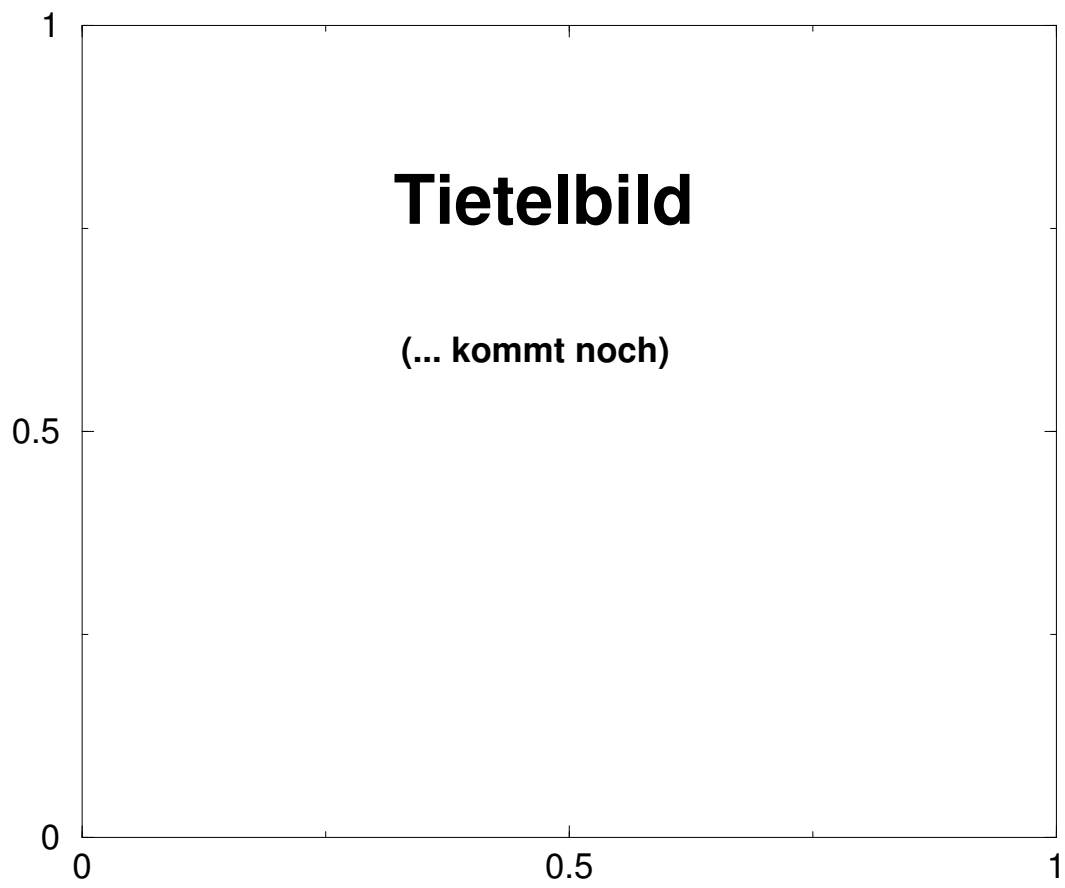
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Extremes and Trend Detection



227 **1. Extreme Value and Trend Analysis based on**
228 **Statistical Modelling of Precipitation Time**
229 **Series**

230 *Silke Trömel and Christian-D. Schönwiese*

231 **Abstract** Application of a generalized time series decomposition technique
232 shows, that observed German monthly precipitation time series can be inter-
233 preted as a realization of a Gumbel distributed random variable with time de-
234 pendent location parameter and time dependent scale parameter. The achieved
235 complete analytical description of the series, that is the probability density
236 function (PDF) for every time step of the observation period, allows prob-
237 ability assessments of extreme values for any threshold at any time. So, we
238 found in the western part of Germany, that climate is getting more extreme in
239 winter. Both the probability for exceeding the 95th percentile and the proba-
240 bility for falling under the 5th percentile are increasing. Contrary results are
241 found in summer. The spread of the distribution is shrinking. But in the south,
242 relatively high precipitation sums become more likely and relatively low pre-
243 cipitation sums become more unlikely in turn of the 20th century.

244 Furthermore, the decomposition technique provides the mean value of the
245 Gumbel distributed random variable for every time step, too. So, an alterna-
246 tive approach for estimating trends in observational precipitation time series
247 is achieved. On that way, the non-Gaussian characteristics can be taken into
248 account and robust estimates can be provided. In contrast, application of
249 the least squares estimator to non-Gaussian climate time series often leads to
250 overestimated trends in the expected value.

◀ **Fig. 1.0.** Application of the generalized time series decomposition technique on modelled time series of the coupled global model ECHAM4/OPYC3 [1.12], 1990-2100, under SRES A2 scenario shows in January in the northern part of Europe strong increases in the probability for exceeding the 95th percentile (top) and less pronounced decreases in the probability for falling under the 5th percentile (bottom). In central Europe an area with increases in both kind of extremes can be seen.

251 1.1 Introduction

252 The analysis of climate variability as reflected in observational records is an
253 important challenge in statistical climatology. In particular, it is important
254 to estimate reliably trends in the mean value as well as changes in the prob-
255 ability of extreme values. In case of monthly or annual temperature data,
256 often Gaussian distributions are adequate for their description. If any linear
257 or non-linear trend of the time series average occurs, this may be described by
258 a shift of a Gaussian probability density function (PDF). Additionally, some
259 other time series components like the annual cycle for example may also vary
260 and cause a shift to higher and lower values again during the observation pe-
261 riod as discussed below [1.2]. Grieser et al. (2002) [1.2] consider temperature
262 time series as a superposition of trends, annual cycle, episodic components,
263 extreme events and noise. Thereby, Gaussian assumptions are used, which im-
264 plies that residuals can not be distinguished from a realization of a Gaussian
265 distributed random variable. However, this simple model of a shifting Gaussian
266 distribution with constant variance over the observation period is not suitable
267 to describe the variability of precipitation time series. We observe a skewed
268 distribution as well as seasonal and sometimes long-term changes in the shape
269 of the distribution. Moreover, changes in the spread of the distribution have
270 to be considered, too. In summary, in addition to the expectation, also the
271 variance and/or further moments may vary with time.

272 Evidently, wrong assumptions concerning the PDF lead also to biased esti-
273 mators of trends or probabilities of extreme values. In consequence, we present
274 a generalized time series decomposition technique which allows any PDF and
275 any related parameter change in time. That is changes in the location, the
276 scale and the shape parameter of a PDF are allowed. In case of Gaussian dis-
277 tributions for example, the location parameter is realized by the average and
278 the scale parameter is realized by the variance.

279 Here, we apply this technique on precipitation time series from a German
280 station network focussed on extreme value and trend analysis. The detection
281 of structured time series components like trends etc., called signals, is based
282 on several parameters of a Gumbel or Weibull distribution, respectively, as
283 described in the following. Section 1.2 provides the definition of the compo-
284 nents for the analysis of monthly climate time series and Sect. 1.3 presents a
285 brief overview of the detection of the analytical functions reflecting the time
286 dependence of the different distribution parameters. For all details see [1.10]
287 and [1.11].

288 **1.2 Components**

The equation

$$S_{j,k}(t) = d_{j,k}t^k \cos\left(2\pi\frac{j}{12}t\right) + e_{j,k}t^k \sin\left(2\pi\frac{j}{12}t\right) \quad (1.1)$$

with wave number $j = 1, \dots, 6$ per year and $k = 0, 1, 2$ gives the basis functions to describe the seasonal component. Evidently, the maximum wave number 6 is tuned to the analysis of climate data with a monthly sampling rate. Besides fixed annual cycles, changes in amplitude and phase are allowed. For the amplitude linear and quadratic time dependence is considered. Superposition of two or three harmonics of the annual cycle with the same wave number j but different time dependence k in one time series yields

$$S_j(t) = A_j(t) \cos\left(2\pi\frac{j}{12}(t - t_j)\right) \quad (1.2)$$

with amplitude

$$A_j(t) = \sqrt{\sum_{k=0}^2 (d_{j,k}^2 + e_{j,k}^2) t^{2k}} \quad (1.3)$$

and phase

$$t_j(t) = \frac{12}{2\pi} \arctan\left(\frac{\sum_{k=0}^2 d_{j,k}t^k}{\sum_{k=0}^2 e_{j,k}t^k}\right). \quad (1.4)$$

289 In this way, the detection of linear, progressive and degressive shaped changes
 290 in phase and amplitude of the annual cycle is possible.

In addition, trends up to the order 5 are considered:

$$T_i(t) = g_i + h_i t^i \quad \text{with } i = 1, \dots, 5. \quad (1.5)$$

291 To detect all significant structures but neglecting, on the other hand, all un-
 292 significant structures, in a first step the detection of the seasonal component
 293 and the trend component of the parameters is performed simultaneously within
 294 the modified stepwise regression procedure (see Sect. 1.3 and [1.9]).

In a second step we observe sometimes relatively low-frequency variations superposed on the components mentioned above. So we offer also polynomial equations up to the order 5:

$$V_l(t) = c_o + \sum_{i=1}^l c_i t^i. \quad (1.6)$$

295 In a third step a search for extreme events, which are independent from changes
 296 in the parameters of the distribution, is performed. According to Grieser et al.
 297 (2002) [1.2], we define extreme events as a relatively small number of extreme
 298 values which are unexpected within the scope of the fitted statistical model.
 299 In contrast to extreme events, extreme values are relatively high or low val-
 300 ues which occur by chance. A positive trend in the mean value, for example,
 301 increases the probability of occurrence of relatively high extreme values.

302 Within the strategy introduced detected extreme events are extracted and
 303 replaced by a random value distributed conform with the PDF and the two
 304 parameters at the given time (see Grieser et al., 2002 [1.2], for further details).
 305 The iterative procedure for the detection of trends, seasonal component and
 306 low-frequency variations is applied until no further extreme events are found.
 307 In the application of Gaussian assumptions and the least-squares method, the
 308 quadratic function which has to be minimized in order to fit any analytical
 309 functions is called distance function of the Gaussian distribution. Generally
 310 speaking, it depends on the robustness ([1.3]) of the applied distance function
 311 whether structured components are more or less influenced by extreme events.

Using an iterative procedure to find within these functions the best model
 equations of two parameters of a PDF leads to a very extensive procedure.
 Note that the model equation of one parameter influences the equation for the
 second parameter and vice versa. To reduce this effort, an additional restriction
 is introduced. For one of the parameters all functions mentioned above are
 offered:

$$P_g(t) = \sum S_{j,k}(t) + \sum T_i(t) + V_l(t). \quad (1.7)$$

The second parameter is assumed to be of minor relevance to describe the time
 series. Only the one cycle per year harmonic and one trend function can be
 chosen:

$$P_r(t) = S_{1,0} + T_i. \quad (1.8)$$

312 At the end of the time series decomposition procedure, a priori assumed resid-
 313 ual distribution is tested. In case of a chosen Gumbel distribution, residuals
 314 should follow a Gumbel distribution $G(0, 1)$ with a location parameter equals
 315 0 and a location parameter equals 1 after elimination of detected structured
 316 components in the parameters $P_g(t)$ and $P_r(t)$. Additional stationarity of the
 317 distribution points to a complete description of the time series within the PDF
 318 and its time-dependent parameters.

319 It depends on the characteristics of the time series under consideration,
 320 which PDF has to be chosen and which PDF parameter implies a larger num-
 321 ber of degrees of freedom in comparison to others. Not before the end of the
 322 procedure, including the analysis of the statistical properties of the residuals,
 323 it can be decided whether the chosen model provides an adequate description
 324 of the time series.

325 **1.3 The distance function and the model selection**
 326 **criterion**

The basis of any time series decomposition technique are the distance function and a model selection criterion. The least-squares estimator broadly used in

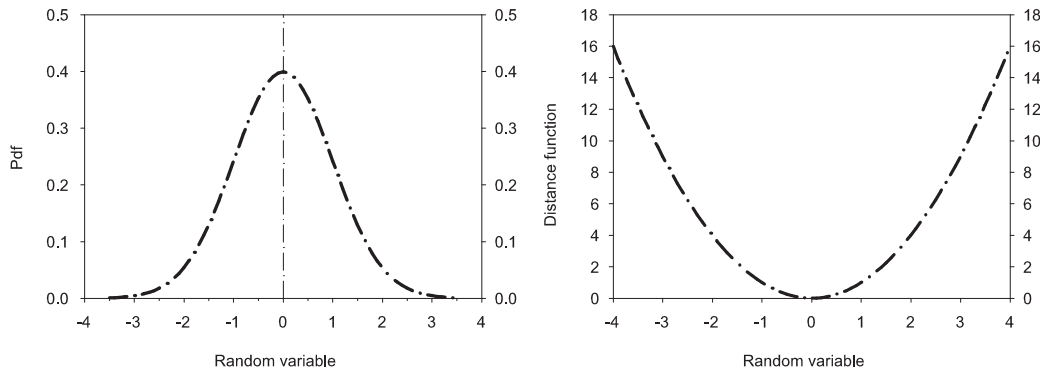


Fig. 1.1. The PDF (left) of the Gaussian distribution and its distance function (right) under the assumption of constant variance.

trend analyses is the maximum likelihood estimator under the assumption of Gaussian distributed residuals with constant variance. This quadratic function which has to be minimized is called distance function of the Gaussian distribution. So, consistently with the maximum likelihood principle another distance function, defined as the negative logarithm of the PDF, replaces the function of squared errors to be minimized, if we choose another distribution as basis of the decomposition procedure, for example the Gumbel or the Weibull distribution. Then time dependence for different distribution parameters can be allowed. With the exchange of the distance function, fitted basis functions describe changes in the location, scale or shape parameter of an appropriate PDF. As an example, the corresponding distance function of the Gumbel distribution is

$$\rho(x, t) = \ln(b(t)) + \exp\left(-\frac{x - a(t)}{b(t)}\right) + \frac{x - a(t)}{b(t)}. \quad (1.9)$$

Under the assumption of statistically independent random variables the coefficients in the model equations for estimating $a(t)$ and $b(t)$ are chosen by minimizing

$$\sum_t \rho(x, t) = \min., \quad (1.10)$$

327 equivalent to the maximum of the loglikelihood function. In this work Powells
 328 method [1.6], p. 406, is used to minimize ρ in the multidimensional space.

Figure 1.1 shows the PDF of the Gaussian distribution and on the right hand-

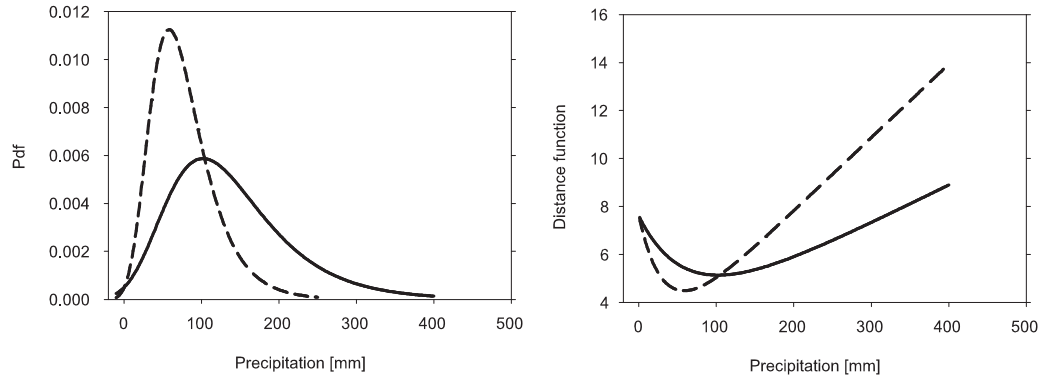


Fig. 1.2. The PDF (left) of the Gumbel distribution with two different location and scale parameters and the associated distance functions (right).

329

side its distance function, the quadratic function. The more deviant the points, the greater the weight. The influence increases very fast because the probability of occurrence of relatively high or relatively low values is very small in the Gaussian case. For comparison the Gumbel distribution for two different location and scale parameters and the associated distance functions are shown in Fig. 1.2. The tails are more prominent and consequently, if we take a look at the distance functions, the influence increases less rapidly. And one value in a given distance from the location estimator has more weight the smaller the scale parameter. So, structured components can be detected in different parameters and estimators of different parameters compete with each other. Finally, Fig. 1.3 shows the Weibull distribution with two different scale and shape parameters. And on the right-hand side the distance functions clearly show the dependence on the shape of the distribution. One important point is that the PDF chosen and the basis functions used to describe the signals are complementary. The other point is, that the basis used to describe one parameter of the PDF influences the basis necessary to describe the second parameter. That is why a dynamic procedure is necessary to estimate the coefficients of the basis functions of the different components of both parameters simultaneously. A flexible model selection criterion often used is the stepwise regression. Stepwise regression [1.9], p. 166, represents a dynamic model selection criterion in order to find the optimal regression equation. Within the generalized time series decomposition a modification to handle two distribution parameters is required.

353

The common iterative application of forward selection and backward elimination is used to determine the model equation of a first distribution parameter as, for example, the scale parameter $b(t)$. But the distance function used now,

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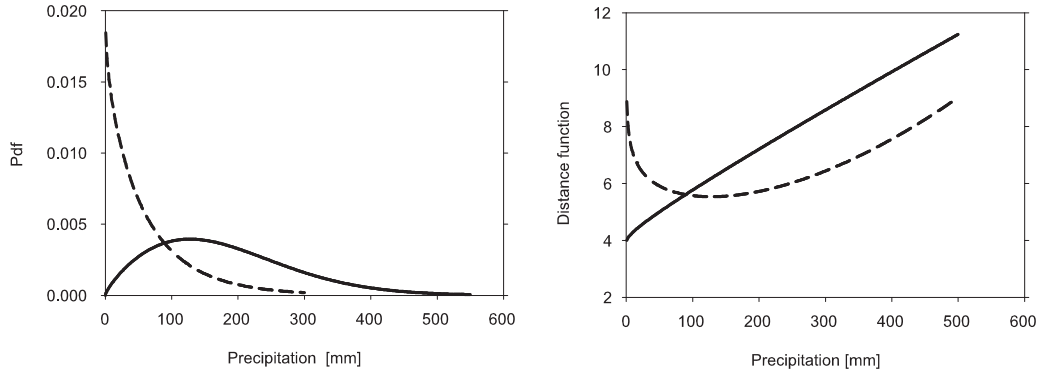


Fig. 1.3. The PDF (left) of the Weibull distribution with two different scale and shape parameters and the associated distance functions (right).

356 also depends on the selected model equation describing the second distribution
 357 parameter (see Eq. 1.9). Between these alternate parts redefinition of the sec-
 358 ond parameter ($a(t)$) is inserted, now taking into account the selected model
 359 of the first parameter ($b(t)$). In the modified version, the flexible strategy of
 360 stepwise regression is used twice. Now the parameters of the model equations
 361 influence themselves mutually. A regressor of the first parameter selected at
 362 an earlier stage can be superfluous because of a new entry candidate in the
 363 model equation of the first parameter or because of the actualisation of the
 364 second parameter and vice versa.

The common F-test statistic broadly used in regression analysis to decide whether a specific regressor contributes significantly to explained variance is sensitive to departures from the Gaussian distribution and, therefore, has to be replaced. A test statistic based on a likelihood ratio test seems to be more applicable, because likelihood values are computed anyhow minimizing the distance function (Eq. 1.10). Define $D(p)$ as the minimum value of (1.10) subject to the model containing p regressors and $D(q)$ as the minimum value of (1.10) subject to the model containing q regressors, with $p > q$, a direct generalization of the common F-test may be based upon the statistic

$$F_M = (D(p) - D(q)) / (p - q) \tag{1.11}$$

365 (Schrader and Hettmansperger, 1980) [1.8] with $p - q$ and $N - p$ degrees of
 366 freedom. Here N represents the number of data. If the density function of
 367 the residuals has not the form $\exp(-\rho)$, the model assumptions are not ful-
 368 filled and a correction term is necessary. For small sample sizes Schrader and
 369 Hettmansperger (1980) propose to compare F_M with a critical value from a
 370 central F distribution.

371 The modified stepwise regression including a modified F-statistic (Eq. 1.11)
 372 represents the basis of a generalized time series decomposition technique. In-

373 serring the corresponding distance function it can be used for time series anal-
 374 ysis based on any data distribution and any statistical model. Thereby, the
 375 initial choice of the probability density function and the corresponding dis-
 376 tance function should depend on the characteristics of the climate variable
 377 under consideration. The Gumbel distribution, the Lognormal distribution as
 378 well as the Gamma distribution are promising distributions already by reason
 379 of the skewness. However, the Weibull distribution seems to be a promising
 380 choice when changes in the shape of the distribution occur. Not until the final
 381 residual analysis at the end of the decomposition tests whether a priori assumed
 382 statistical model can be confirmed. If so, a complete analytical description of
 383 the time series is achieved.

384 Actually, the generalized time series decomposition technique in a deter-
 385 ministic and a statistical part is applied to 4 different models: Observed time
 386 series can be interpreted as

- 387 • a realization of a Gaussian-distributed random variable with time-dependent
 388 location (mean value) and additional time-dependent scale parameter (stan-
 389 dard deviation),
- 390 • a realization of a Gumbel-distributed random variable with time-dependent
 391 location and scale parameter, it depends on the choice to offer the location
 392 or the scale parameter the greater pool of regressors if we talk about
 393 – the Gumbel model with special emphasize on location or
 394 – the Gumbel model with special emphasize on scale.
- 395 • a realization of a Weibull-distributed random variable with time-dependent
 396 scale and shape parameter.

397 1.4 Application to a German station network

398 1.4.1 General remarks

The generalized time series decomposition technique is applied now to monthly precipitation sums from a German station network of 132 time series covering 1901-2000. At least for the most part of the sample the decomposition technique shows, that observed time series can be interpreted as a realization of a Gumbel distributed random variable with time-dependent location parameter $a(t)$ and time-dependent scale parameter $b(t)$. So the decomposition is based on the probability density function (PDF) of the Gumbel distribution

$$f(x, t) = \frac{1}{b(t)} \left\{ \exp \left(-\frac{x - a(t)}{b(t)} \right) \exp \left[-e^{-(x-a(t))/b(t)} \right] \right\}. \quad (1.12)$$

399 Since the Gumbel model with emphasis on scale leads to a better description
 400 of the time series, we define the location parameter $a(t) = P_r(t)$ and the scale

parameter $b(t) = P_g(t)$ (see Eqs. 1.7 and 1.8). However, 7 out of 158400 (132-
 1200) monthly precipitation sums are extracted as extreme events which are
 unexpected within the scope of the fitted Gumbel model. As aforementioned
 the analysis of the remaining residuals represent an important part of the
 analysis procedure. After elimination of the detected structures in the location
 and in the scale parameter, residuals should fulfill the condition of the a priori
 assumed statistical model. Concerning the German station network of 132
 monthly precipitation sums, the residuals should be undistinguishable from the
 realization of a Gumbel distributed random variable $G(0,1)$ with the location
 parameter 0 and the scale parameter 1. In fact the Kolmogorov-Smirnov test
 [1.6] rejects in 7 out of 132 cases this hypothesis with a probability larger than
 90%. This is less ($<10\%$) than may be expected by chance. Furthermore, again
 a Kolmogorov-Smirnov test statistic is used in order to check the stationarity
 of the residuals. Now in 6 out of 132 cases stationarity is rejected with a
 probability larger than 90%. Consequently, the residuals confirm the model
 applied and a complete description of the observed time series on the basis of
 the Gumbel model with emphasis on the scale could be achieved.

The provided PDF for every time step of the observation period allows prob-
 ability assessments of extreme values, i.e. the probability for exceeding a given
 precipitation sum (threshold) at any time. In this context it should be reem-
 phasized that in contrast to analyses dealing with the familiar Generalized
 Extreme Value distribution [1.4] [1.1], the generalized time series decomposi-
 tion technique presented in this issue requests the analytical description of the
 whole times series instead of addressing exclusively the maxima of the observa-
 tions. Furthermore, with the PDF the mean value of the Gumbel distributed
 random variable can be given for every time step, too.

In Section 1.4.2 the time series decomposition of an exemplary time series is
 discussed in more detail for a better understanding of the method introduced.
 Subsequently, results concerning changes in the probability of extreme values
 and the expected value of the entire station network are presented in Sect. 1.4.3
 and Sect. 1.4.4.

1.4.2 Example: Eisenbach-Bubenbach

In case of the Gumbel model with emphasis on scale the greater pool of re-
 gressors is offered to the scale parameter $b(t)$ of the Gumbel distribution and
 the smaller one to the location parameter $a(t)$. Table 1.1 shows the significant
 functions detected in the location parameter $a(t)$ and the scale parameter $b(t)$
 of the Gumbel distribution to describe the time series observed in Eisenbach-
 Bubenbach (47.97°N , 8.3°E). Phase angles are defined with reference to De-
 cember 15th. Consequently, we observe a superposition of two harmonics with
 wave number one per year. The function $S_{1,0}$ has a constant amplitude with
 maximum in September but $S_{1,1}$ reveals linear time dependence in the ampli-

Table 1.1. Significant functions in the location parameter $a(t)$ and scale parameter $b(t)$ for the description of the monthly precipitation time series observed in Eisenbach-Bubenbach.

Parameter	Function	Amplitude [mm]	Phase [days]
$b(t)$	$S_{1,0}$	$7.04 \cdot t^k$	-82.73
	$S_{1,1}$	$0.148 \cdot 10^{-1} \cdot t^k$	44.87
	T_1	5.78	$-$
$a(t)$	$S_{1,0}$	$-8.89 \cdot t^k$	24.21

442 tude and has its maximum in January. So, the winter becomes more variable
 443 and the summer becomes less variable during the observation period. The sea-
 444 sonal component in the scale parameters shows a phase shift of 109 days from
 445 September to January. The resulting effects can also seen in Figs. 1.4 and 1.5.
 446 The former figure illustrates the decrease in the scale parameter in August. The
 447 functions given in Tab. 1.1 define the scale and the location parameter of the
 Gumbel distribution for every time step. No low-frequency variations ($V_i(t)$)

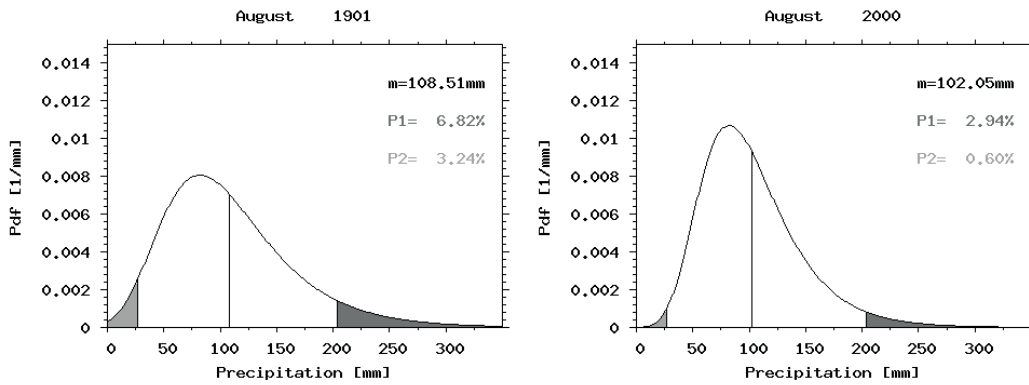


Fig. 1.4. On the basis of the entire time series estimated PDFs for two time steps: August 1901 (left) and August 2000 (right). Areas below the 5th percentile are marked in light grey, the areas above the 95th percentile in dark grey and the vertical line marks the expected value m . The respective value m , the probability $P1$ for exceeding the 95th percentile and the probability $P2$ for falling under the 5th percentile are given in the upper right corner respectively.

448 and no unexpected values defined as extreme events (see Sect. 1.2) within the
 449 time series decomposition technique can be detected in the time series. How-
 450 ever, probability assessments of extreme values can be based on the analytical
 451 description provided. Figure 1.4 shows the estimated PDFs for the first and the
 452 last August in the observation period. The probability $P1$ for exceeding the
 453 95th percentile, in this case 204 mm, and the probability $P2$ for falling under
 454 the 5th percentile, in the present case 27 mm, are also given for the two time
 455 steps. The corresponding areas within the PDF are marked in dark grey and
 456 light grey, respectively. First of all we see is a decrease in the scale parameter,
 457

458 going along with a decrease in the probability of exceeding the 95th percentile
 459 from 6.8% to 2.9% and we see a smaller decrease in the probability of falling
 460 under the 5th percentile from 3.2 to 0.6%. The probability of both kinds of
 461 extremes has decreased and in contrast to the Gaussian model the expected
 462 value is also affected. The expected value decreases from 109 mm to 102 mm in
 463 the observation period. It is worth to mention that the least squares estimator
 464 is not able to describe those decreases or increases in variability and not the
 influence on the expected value either. In Fig. 1.5 integration over the dark

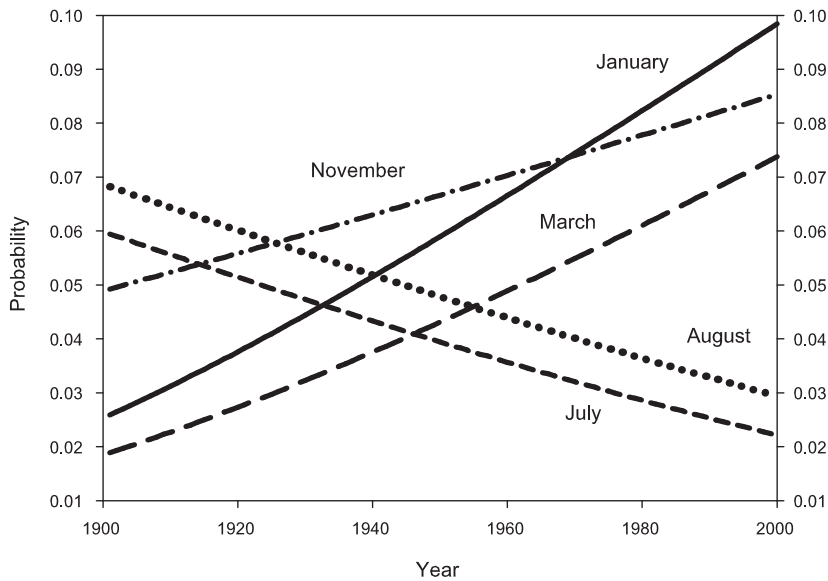


Fig. 1.5. Estimated probabilities for exceeding the 95th percentile between 1901 and 2000 in January, March, July, August and November at the station Eisenbach-Bubenbach.

465 grey area for every time step and different months has been done. Contrary
 466 tendencies in the probability of exceeding the 95th percentile can be seen. At
 467 the beginning the relatively high precipitation sums occurred with higher prob-
 468 ability in summer but at the end of the observation period the probability
 469 for exceeding the threshold is highest in January. Both, the phase shift in the
 470 seasonal component and the positive linear trend detected in the scale param-
 471 eter describe an increasing variability in winter. The distribution is widening.
 472 The tendencies in summer are contrary. Both kind of extremes become more
 473 unlikely between 1901 and 2000.
 474

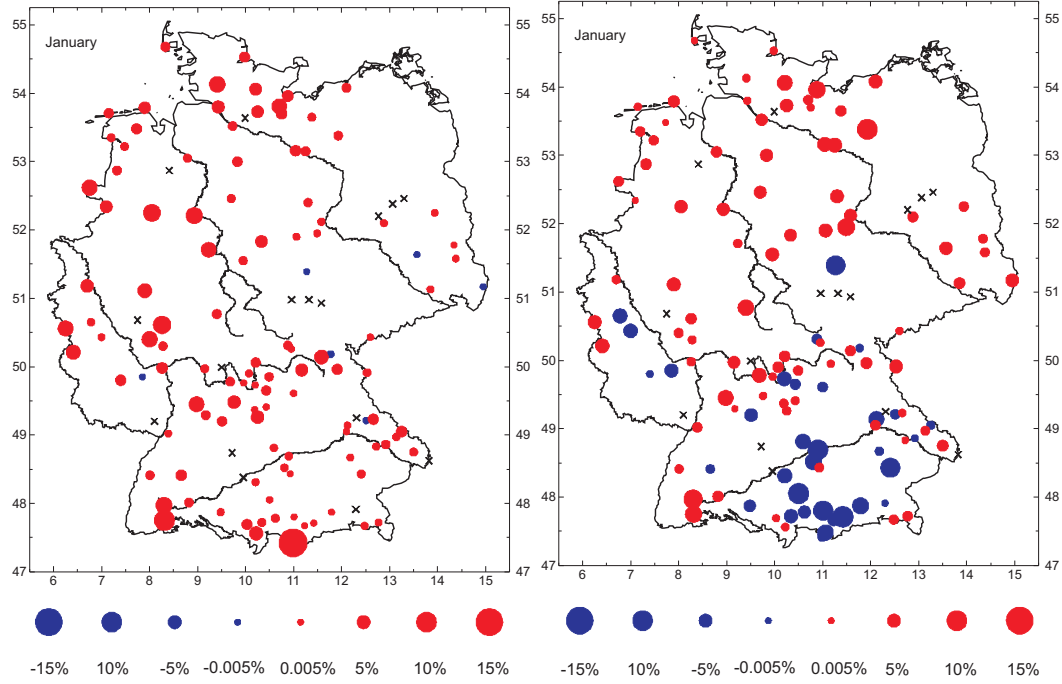


Fig. 1.6. Changes in the probability of occurrence of a monthly precipitation sum greater than the 95th percentile (left) and in the probability of occurrence of a monthly precipitation sum smaller than the 5th percentile (right). Results are given for January in the observation period 1901-2000. Red dots indicate an increase and blue dots indicate an decrease in the probability of occurrence. The size of the dots is proportional to the magnitude in probability change.

475 **1.4.3 Probability assessment of extreme values**

476 If the time series decomposition in a statistical and a deterministic part suc-
 477 ceeded, the time-dependent PDF $f(x, P_g(t), P_r(t))$ as a complete analytical
 478 description is provided (see again Fig. 1.4).

In this model

$$\begin{aligned}
 p_G(x \geq x_s, t) &= 1 - \int_{-\infty}^{x_s} \frac{1}{P_g(t)} \left\{ \exp\left(-\frac{x - P_r(t)}{P_g(t)}\right) \exp\left[-e^{-(x - P_r(t))/P_g(t)}\right] \right\} dx \\
 &= 1 - \exp\left\{-\exp\left(-\frac{x_s - P_r(t)}{P_g(t)}\right)\right\}
 \end{aligned} \tag{1.13}$$

gives the probability for exceeding the threshold x_s at time t . Significant structured components in the location and the scale parameter cause changes in the probability of occurrence of these high precipitation sums. The probability for falling under x_s is given by

$$\begin{aligned}
 p_G(x \leq x_s, t) &= \int_{-\infty}^{x_s} \frac{1}{P_g(t)} \left\{ \exp\left(-\frac{x - P_r(t)}{P_g(t)}\right) \exp\left[-e^{-(x - P_r(t))/P_g(t)}\right] \right\} dx \\
 &= \exp\left\{-\exp\left(-\frac{x_s - P_r(t)}{P_g(t)}\right)\right\}.
 \end{aligned}
 \tag{1.14}$$

479 In Fig. 1.6 tendencies in the probability of a monthly precipitation total greater
 480 than the 95% percentile (left map) and tendencies in the probability of a
 481 monthly precipitation total smaller than the 5% percentile (right map) are
 482 shown. That is for every single time series the threshold is selected so that
 483 95% (5%) of all monthly rainfall totals of the series are smaller than the re-
 484 spective threshold. The result is in January an increase in the probability of
 485 exceedance in the overwhelming majority of stations. In the right map the
 tendencies in the probability of a monthly precipitation total less than the

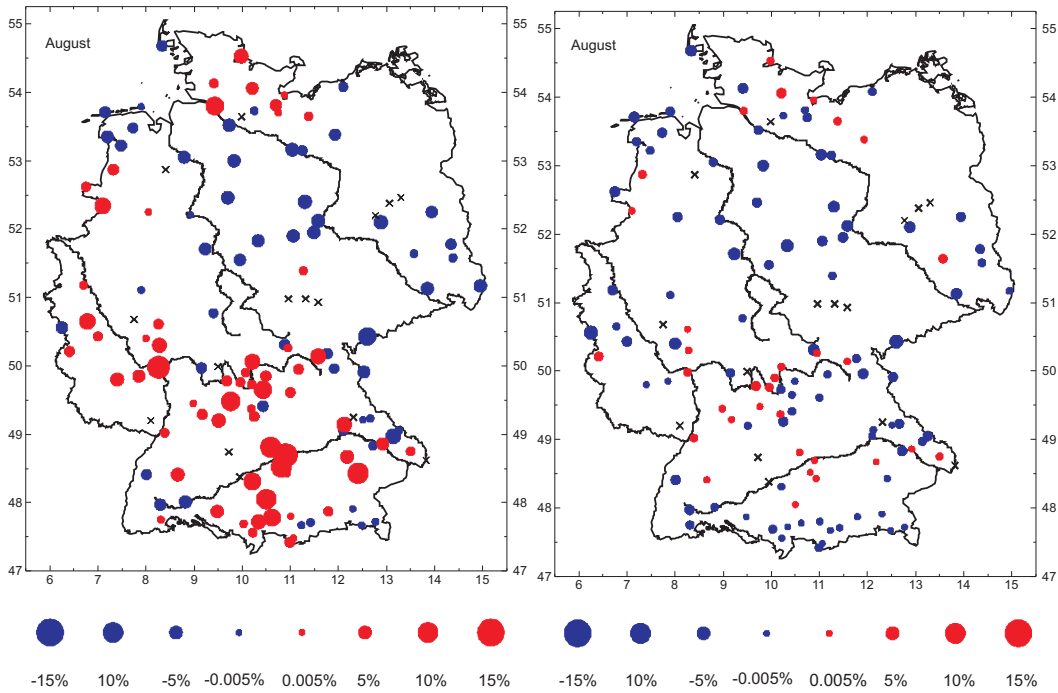


Fig. 1.7. Analogous maps to Fig. 1.6 with results for August.

486
 487 5th percentile are seen. In January we see in the northern part of Germany
 488 increases in the probability of occurrence for these small precipitation totals,
 489 too. So relatively high and relatively small precipitation totals become more
 490 likely during the 20th century. The distribution is widening. However, in the
 491 south the distribution is shifting to higher values.

492 Figure 1.7 shows the respective results in August. In the northern part of
 493 Germany decreases in the probability of relatively high precipitation sums are
 494 detected during the 20th century. But in the south we see several increases

495 in August as well as in January. In the right map it can be seen, that de-
 496 creases in the probability of occurrence of relatively small precipitation totals
 497 are detected in the overwhelming majority of stations. Combined with the ten-
 498 dencies in the probability of exceedance of the 95th percentile we come to the
 499 conclusion that in summer in the northern part of Germany the extreme high
 500 and extreme low precipitation totals are getting more unlikely. In the south
 501 we observe again at several stations a shift of the distribution to higher values
 502 but results are not as uniform in that region in summer.

503 1.4.4 Changes in the expected value

As could already be seen in Fig. 1.4 the statistical modelling provides the expected value for every time step of the observation period. Consequently, the method introduced represents an alternative approach to estimate trends. The difference in the expected value between the January 2000 and the January in the year 1901 is defined as the trend in the expected value at every station in that month. With the use of an appropriate distance function relatively high precipitation sums do not get more weight than can be justified from a statistical point of view. Additionally, changes in different parameters of the distribution can be taken into account, because the expected value of a Gumbel distributed random variable is defined as

$$\mu(t) = a_G(t) + b_G(t)\gamma \approx a_G(t) + 0.57722 \cdot b_G(t) \quad (1.15)$$

with the location parameter $a_G(t)$, the scale parameter $b_G(t)$, and Eulers constant γ . Because of the skewness of the distribution a change in the scale parameter causes changes in the expected value, too. And the expected value of a Weibull distributed random variable depends on the location parameter a_W , the scale parameter b_W and the shape parameter c_W :

$$\mu(t) = a_W(t) + b_W(t)\Gamma\left(1 + \frac{1}{c_W}\right). \quad (1.16)$$

The Gamma-function Γ is defined as

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt. \quad (1.17)$$

504 A familiar nonparametric trend test would be the Mann-Kendall trend test
 505 [1.5]. Even though this test contains information about a increase or decrease
 506 in the time series considered, no information about the temporal evolution
 507 nor the amplitude of the trend is provided. In most cases, changes in the
 508 expected values of non-Gaussian climate time series, e.g. ordinary trend maps
 509 are estimated using the least squares method, too. Comparison of the trend

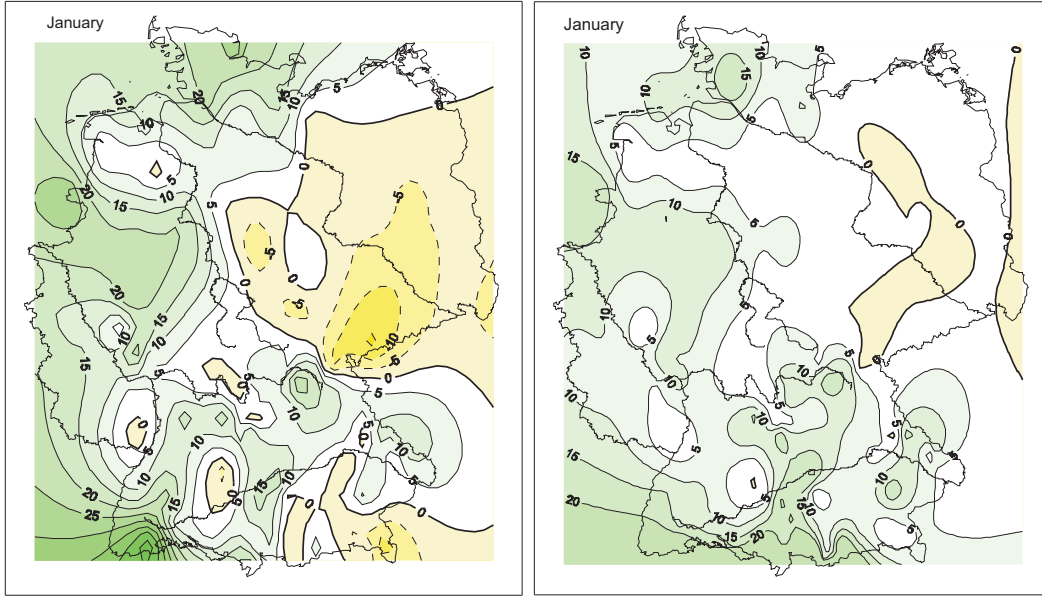


Fig. 1.8. Observed trends 1901-2000 in January estimated with the least-squares method (left) and on the basis of the statistical modelling of the time series using the distance function of the Gumbel distribution (right).

510 map on the basis of the least-squares-method with the trend map estimated on
 511 the basis of the method introduced (Fig. 1.8) shows in January similar spatial
 512 structures. We observe increases in the western and southern part of Germany,
 513 but decreases in the eastern part. However, the amplitudes are smaller using
 514 the distance function of the Gumbel distribution.

515 Another advantage of the method introduced is that trends of a particu-
 516 lar month are estimated on the basis of the whole sample size. In order to
 517 demonstrate the large effect of the sample size on trend analysis results, Monte-
 518 Carlo simulations have been performed. 100 time series with a length of 100
 519 or 1 000 time steps, respectively, of Gumbel-distributed variables with trends
 520 $\Delta\mu$ have been generated. Subsequently, we applied the least-squares estimator
 521 and tried to find the prescribed trend. In Tab.1.2 the mean least-squares-
 522 estimator $\overline{\Delta\mu}_{KQ}$, its standard deviation $\sigma_{\Delta\mu}$, as well as the smallest and the
 523 greatest estimator, $\Delta\mu_{KQ}^-$ and $\Delta\mu_{KQ}^+$ are shown for eight different experiments
 524 using the longer generated time series (1 000 time steps). A positive bias can
 525 be observed. Table 1.3 shows the analogous results for the smaller sample size
 526 (100 time steps). The absolute value of the bias has clearly increased but the
 527 trend estimator shows a negative bias now. The estimators variance $\sigma_{\Delta\mu}$ has in-
 528 creased remarkably. It may be higher than the trend magnitude. Consequently,
 529 the greater sample size of precipitation totals taken into account within the
 530 statistical modelling approach (1 200 values instead of 100 in the present case)
 531 implies a smaller mean squared error of the maximum likelihood estimator
 532 given by the sum of the quadratic bias and the variance of the estimator.

Table 1.2. Trends of 100 generated Gumbel-distributed time series with 1000 time steps and different linear changes in the location parameter (Δa) and constant or additional changing scale parameter (b). The resulting changes in the expected value $\Delta\mu$ (see again Eq. 1.15) are compared to mean least-squares estimator of the 100 time series $\overline{\Delta\mu}_{KQ}$, the standard deviation $\sigma_{\Delta\mu}$ of the estimator as well as the greatest and the smallest trend estimator ($\Delta\mu_{KQ}^+$ bzw. $\Delta\mu_{KQ}^-$).

	$\Delta a=15$ b=50	$\Delta a=15$ b=20	$\Delta a=8$ b=40	$\Delta a=0$ b=40	$\Delta a=8$ b=40+ $\Delta 8$	$\Delta a=-15$ b=50	$\Delta a=-8$ b=40- $\Delta 8$	$\Delta a=0$ b=40+ $\Delta 10$
$\Delta\mu$	15	15	8	0	12.6	-15	-12.6	5.77
$\overline{\Delta\mu}_{KQ}$	15.49	15.20	8.39	0.39	13.05	-14.51	-12.27	6.22
$\sigma_{\Delta\mu}$	6.97	2.79	5.57	5.57	6.16	6.70	5.02	6.31
$\Delta\mu_{KQ}^-$	0.78	9.31	-3.38	-11.38	0.08	-29.22	-22.83	-7.06
$\Delta\mu_{KQ}^+$	33.44	22.38	22.75	14.75	28.23	3.44	1.27	21.81

Table 1.3. Analogous to Tab. 1.2, but considering Gumbel-distributed time-series with 100 time steps.

	$\Delta a=15$ b=50	$\Delta a=15$ b=20	$\Delta a=8$ b=40	$\Delta a=0$ b=40	$\Delta a=8$ b=40+ $\Delta 8$	$\Delta a=-15$ b=50	$\Delta a=-8$ b=40- $\Delta 8$	$\Delta a=0$ b=40+ $\Delta 10$
$\Delta\mu$	15	15	8	0	12.6	-15	-12.6	5.77
$\overline{\Delta\mu}_{KQ}$	12.85	14.14	6.28	-1.72	10.78	-17.15	-14.21	3.90
$\sigma_{\Delta\mu}$	23.82	9.53	19.06	19.06	20.91	23.82	17.31	21.39
$\Delta\mu_{KQ}^-$	-43.84	-8.54	-39.07	-47.07	-38.96	-73.84	-55.18	-49.93
$\Delta\mu_{KQ}^+$	59.52	32.81	43.62	35.62	51.44	29.52	22.13	46.37

533 **1.5 Conclusions**

534 A generalized consistent decomposition procedure of precipitation time series
 535 into a statistical and a deterministic part is introduced. The basis functions
 536 allowed to describe the deterministic components only contain trends, annual
 537 cycle, episodic component and extreme events in order to restrict to physically
 538 explainable functions. Under the additional assumption that climate change
 539 is not restricted to the mean value the signal detection technique is applied
 540 to two instead of one parameter of a PDF, which can be chosen without any
 541 further restriction.

542 In particular, we show that a time series decomposition technique based on
 543 a Gumbel distribution, with flexible location and scale parameter, succeeds to
 544 describe monthly precipitation total time series from German stations com-
 545 pletely. The model provides a full analytical description of the time series. On
 546 this basis probabilities of exceeding defined thresholds can be estimated reli-
 547 ably for every time step of the observation period. But the provided complete

analytical description can be used to calculate the expected value for every time step, too. So, statistical modelling represents an alternative approach for estimating broadly used trends in observational precipitation time series. In this way non-Gaussian characteristics can be taken into account. Additionally, changes in different parameters can be considered and the mean squared error of the trend estimator is smaller using the statistical modelling.

Application of the method to a German station network of 132 time series covering 1901-2000, shows in winter and summer at several stations in the southern part of Germany an increase in the probability of exceeding the 95th percentile and a decrease in the probability of falling under the 5th percentile. In the western part, we observe the same phenomenon in the summer months, but these changes go along with smaller magnitudes. However, climate is getting more extreme in that region in winter: Probability for both exceeding the 95th percentile and for falling under the 5th percentile is increasing. In the eastern part of Germany, increases in the probability of occurrence of relatively low precipitation in winter as well as decreases in both probabilities (>95th percentile, <5th percentile) in summer and autumn prevail.

Exemplary, the trend map on the basis of the familiar least squares method is compared to the trend map calculated on the basis of the Gumbel model. Both maps show positive trends in the western and southern part of Germany and negative trends in the eastern part. However, the robust method provides smaller amplitudes.

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ACF	autocorrelation function
AP1824	Area Precipitation 1824
ASA	autocorrelation spectrum analysis
CDF	cumulative distribution function
CMDS	classical multidimensional scaling
CWT	continous wavelet transformation
DEM	Digital Elevation Model
DJF	December, January, February
DFA	detrended fluctuation analysis
DWD	Deutscher Wetterdienst (German Weather Service)
ECHAM	
ENSO	El Niño/Southern Oscillation
EOF	empirical orthogonal function
EXP	exponential distribution
599 FA	(standard) fluctuation analysis
FARIMA	fractionally integrated auto-regressive moving average process
FAR	fractionally integrated auto-regressive process
FMA	February, March, April
FFT	fast Fourier transform
FFM	Fourier Filtering Method
GEV	generalised extreme value distribution
GLO	generalised logistic distribution
GNO	generalised normal distribution
GPD	generalized Pareto distribution
GRDC	Global Runoff Data Centre
HadRM	Hadley Centre regional model
i.i.d.	independent and identically distributed
ISOMAP	isometric feature mapping
JJA	June, July, August

LARSIM	Large Area Runoff Simulation Model
LRD	long-range dependence
MC	Monte-Carlo simulations
MICE	modelling impact of climate change
MDS	multidimensional scaling
ML	maximum likelihood
MLE	maximum likelihood estimation
MRL	mean residual life plot
M-SSA	multichannel singular system analysis
$\mathcal{N}(\mu, \sigma^2)$	Gaussian/normal distribution with mean μ and variance σ^2 .
N	Precipitation
NAO	North Atlantic Oscillation
NLDR	nonlinear dimensionality reduction
OND	October, November, December
PCA	principal component analysis
PC	principal component
PDF	probability density function
PE3	Pearson type-III distribution
POT	peak over threshold
PP	point process
PUB	prediction of ungauged basins
PWM	probability weighted moments
R	environment for statistical computing www.r-project.org
RC	reconstructed component
ROI	
SLP	see level pressure
SRD	short-range dependence
SSA	singular system analysis
t	air temperature
u	wind speed
UNF	uniform distribution
$W\mathcal{N}(\mu, \sigma^2)$	Gaussian/normal white noise
WAK	Wakeby distribution
WTMM	
d	Wind Direction
m a.s.l.	meter above sea level

601 **Subject Index**

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 604 appears in a title or subheading. Page numbers that are marked in *italics*
 605 indicate that the entry appears in the caption of a table or figure.

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