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Probability change of extreme precipitation observed from 1901 to 2000 in Germany

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With 13 Figures

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Summary

A generalized time series decomposition technique is applied to monthly total precipitation data from a German station network of 132 time series covering 1901–2000. The decomposition technique shows that observed time series can be interpreted as a realization of a Gumbel distributed random variable with time-dependent location parameter and time-dependent scale parameter. It provides a full analytical description of the series in terms of the probability density function (PDF) for every time step of the observation period. Consequently, probability assessments of extreme values are possible for any threshold at any time.

Most of the year, an increase in the probability of exceeding the 95th percentile and a decrease in the probability of falling under the 5th percentile can be detected at several stations in the southern part of Germany. In the western part, we observe the same phenomenon in the summer months, but these changes go along with smaller magnitudes. However, climate is getting more extreme in that region in winter: Probability for both exceeding the 95th percentile and for falling under the 5th percentile is increasing. In the eastern part of Germany, increases in the probability of occurrence of relatively low precipitation in winter as well as decreases in both probabilities (>95th percentile, <5th percentile) in summer and autumn prevail.

1. Introduction

In addition to the consideration of climate changes concerning the mean value of climate elements, considerable interest has emerged in the probability of occurrence of extreme events which can have major impacts on society, the economy, and the environment. So, an interest to estimate changes in the probability of occurrence of extreme values is obvious.

In the context of statistical analysis, Gaussian distributions have many convenient properties. Consequently, climate time series are often assumed to be normal. As stated in the central limit theorem (Storch and Zwiers, 1999) it can be a good approximation, but the theorem makes an asymptotic statement. Neglecting different rates of convergence for different climate variables often leads to unjustified assumptions and biased estimators. For variables that are not well approximated by normal distributions, like precipitation, changes in the mean value can be accompanied by changes in the spread of the distribution (scale parameter) or in the shape of the distribution. Note that Katz and Brown (1992) stressed the importance of the variability for the probability of extremes. As an example, Fig. 1 shows the stationary fitted Gumbel distribution of observed monthly total precipitation at Eppenrod (50.4° N, 8.0° E), a station in the western part of Germany, for the observation period 1901–1950 and 1951–2000, respectively. A shift of the distribution, e.g. an increase in the location parameter, and a slight increase in the scale parameter

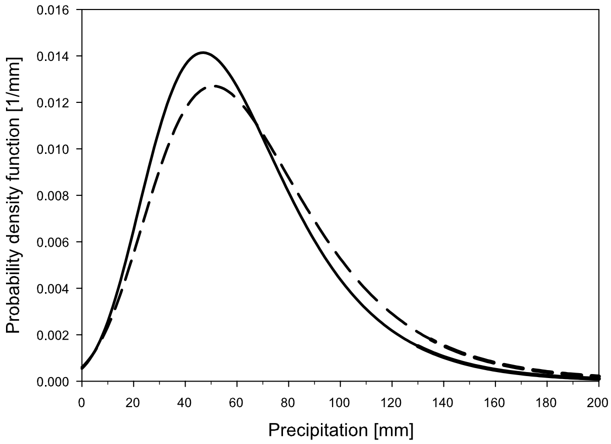

Fig. 1. Comparison of the stationary Gumbel distribution fitted to monthly total precipitation observed 1901–1950 (solid line) and 1951–2000 (dashed line), respectively, in Eppenrod (50.4° N, 8.0° E)

can be observed at this station. But these results are based on the assumption of stationarity for the first and second 50 years of the time interval considered.

However, in some regions tendencies in summer and winter seem to be opposite (Trömel, 2004) and the assumption of stationarity (Fig. 1) makes only minor average changes visible. An analysis of the time series of every single month would handle the seasonal dependence. In order to estimate the temporal development in the probability of occurrence of extreme events for each month using a shifting time window statisticians encounter a dilemma. On the one hand, the fit of a distribution should be based on a sufficient large sample size, but on the other hand they are interested in a high temporal resolution.

Fourier transformation (Schlittgen and Streitberg, 1999) or wavelet decomposition techniques (Mallat, 1997), for example, yield a complete decomposition of time series using orthonormal basis functions. On the contrary, Grieser et al. (2002) decompose surface air temperature into a structured component and stationary Gaussian distributed noise. Superposition of detected components gives the time-dependent average of the assumed Gaussian probability density function. They only use a limited basis which can be interpreted from a climatological point of view. Therefore non-orthogonal basis functions are accepted. The signal is defined as a sum of a constant or significantly changing annual cycle, trends, extreme events, episodic and harmonic components. The PDF for every time-step of the sampling time enables probability assessments of extreme values for any threshold and any time on the basis of the whole data set. The estimation of the probability of extreme events on the basis of the whole data set is in contrast to analyses with the generalized extreme value distribution (GEV) (Gumbel, 1958; Leadbetter et al., 1983; Dupuis and Field, 1998). The latter is an asymptotic theory to describe the maximum of the sample.

Trömel and Schönwiese (2005) introduced a generalized time series decomposition technique in order to gain a statistical modelling of non-Gaussian climate time series. The appendix outlines this method. A successful decomposition provides a complete analytical description of the series, e.g. the probability density function (PDF) for every single time step of the observation period can also be achieved for precipitation time series. This statistical tool is applied to a German station network of 132 time series of monthly total precipitation covering 1901–2000. The time series are most likely homogeneous up to 1990 and updated without any further homogeneity tests. According to Schönwiese and Rapp (1997) most likely homogeneous means here that at least 3 out of 5 different homogeneity tests decided to homogeneity instead of nonhomogeneity. Anyhow, discontinuities of precipitation measurements or different orientation and height of precipitation gauges are likely included in the time series and may alter measured precipitation especially in case of snow (see again Schönwiese and Rapp, 1997). Resulting errors are hard to quantify and are not considered in this analysis. Nevertheless, the spatial structures found in the probability of occurrence of extreme precipitation sums are most likely not caused by these errors. Interpretation of the time series as a realization of a Gumbel distributed random variable with time-dependent location parameter and additional time-dependent scale parameter was successful (see again Trömel (2004), Trömel and Schönwiese (2005)). The residual analysis (see appendix) represents an objective statistical test whether the chosen model provides one possible description. Furthermore, the interpretation as a realization of a Gumbel distributed random variable is favoured against the interpretation as a realization of a Weibull distributed random variable (using the basis functions given in the appendix). In this paper the results of the statistical modelling are used to estimate observed changes in the probability of extremes. The interpretation of every single observational precipitation sum as a realization of the PDF at this time gives via integration over the area of interest the possibility to quantify the probability of exceeding optional upper or lower thresholds, respectively, for any time step of the observation period. Every single PDF is estimated on the basis of the whole time series. This approach allows one to take into account instationarity of observational precipitation time series, particularly with regard to changes in different parameters (location parameter, scale and shape parameter) of a non-Gaussian PDF.

2. Results

By means of the generalized time series decomposition technique we can now have a closer look on the changes in the precipitation time series observed in Eppenrod. Figure 2 compares the estimated PDF in January 1901 and January 2000. Additionally, the probability P1 for exceeding the 95th percentile and the probability P2 for falling under the 5th percentile are given for these two time steps. In this paper the percentiles are defined for all months taken together for any time series, respectively. Consequently, 5% of all monthly precipitation data of a time series are smaller than the 5th percentile and 5% are larger than the 95th percentile. Obviously, changes in January are more pronounced than supposed on the basis of Fig. 1. The probability of occurrence of relatively high precipitation has considerably increased from 1.5% to 8.4%, see Fig. 2. But the positive trend in the scale parameter causes a simultaneous increase in the probability of occurrence of relatively low precipitation, too. The probability P2 has increased from 3% to 5.2% during the 20th century. It is worth mentioning that the common least-squares estimator is not able to describe both, such increases in variability and the effect on the expected value m (see again Fig. 2).

Additionally, Fig. 3 shows the probability of exceeding the 95th percentile for the entire observation period and compares the temporal development in January with observed changes in March, July, August and November. Increases in the probability of exceedance are most pronounced in winter, but we also observe increases in spring and autumn at Eppenrod. However, in July and August the probabilities remain nearly constant during the 20th century. For the interpretation of the results presented in this Figure and the following ones as well, the definition of the percentiles should be taken into account. On the one hand the annual cycle of precipitation leads to a negative bias of absolute changes in the probability of exceedance in the low precipitation months and leads to a positive bias in the high precipitation months. On the other hand the definition of percentiles for each calendar month separately is not appropriate for the absolute comparison of the different months. In Eppenrod, for example, the highest precipitation sums should be expected in August at the beginning of the 20th century, but at the end of the 20th

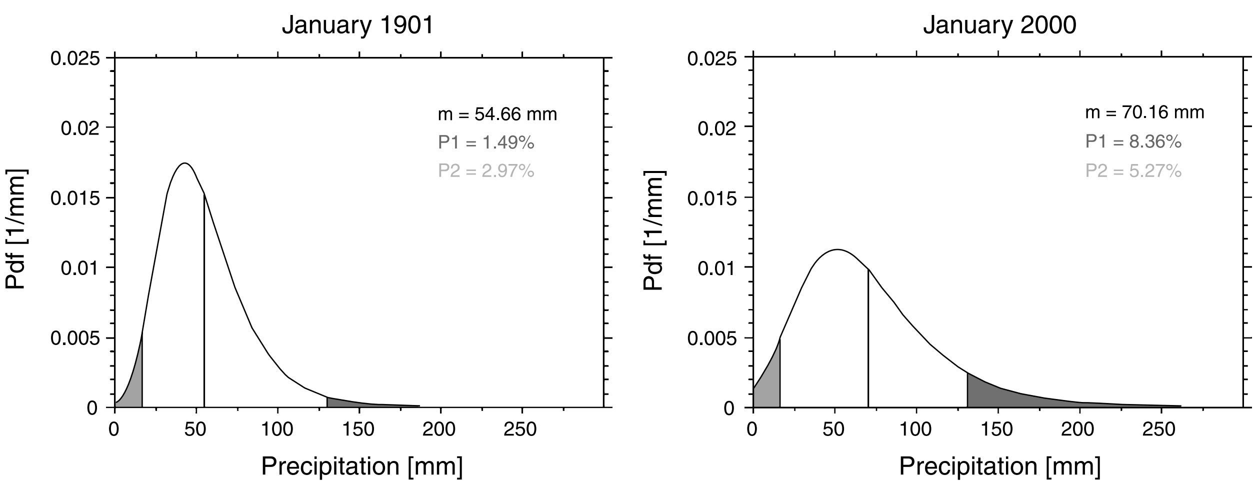

Fig. 2. Two snapshots of the analytical description (PDF) of the monthly precipitation time series observed in Eppenrod, Germany. Integration gives the probability P1 for exceeding the 95th percentile (marked in dark grey) and the probability P2 for falling under the 5th percentile (marked in light grey). The vertical black line shows the expected value m at a given time

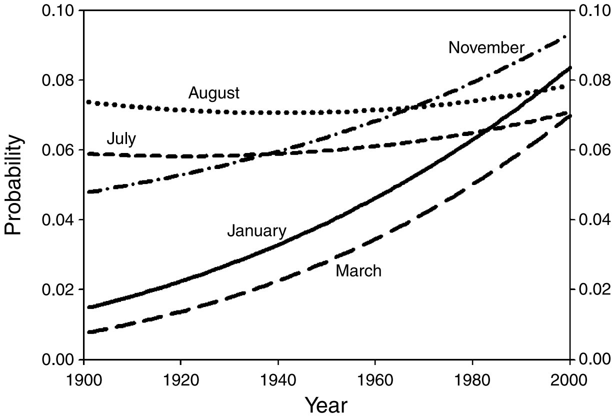

Fig. 3. Development of the probability of exceeding the 95th percentile in January, March, July, August and November at the Station Eppenrod

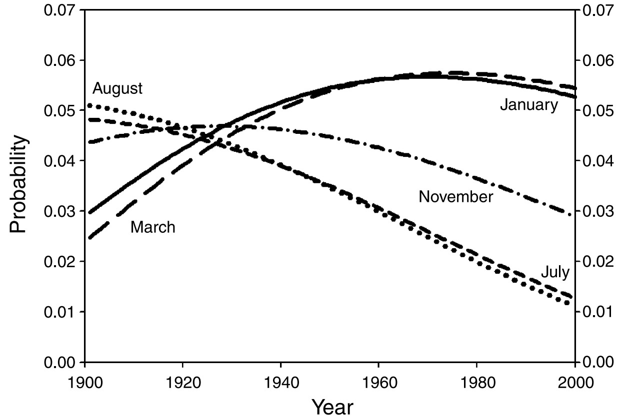

Fig. 4. Development of the probability for falling under the 5th percentile in January, March, July, August and November at the Station Eppenrod

century they occurred in November. Analogous to Fig. 3, the temporal development in the probability of falling under the 5th percentile are shown for the same months in Fig. 4. In January and March the probability of occurrence for relatively low precipitation increased during the 20th century, too. So, the climate has become more extreme. But in July and August, relatively low precipitation has become more unlikely. In November the distribution is shifted to higher precipitation, going along with an increase in the probability of occurrence of relatively high precipitation and a decrease in the occurrence of low values. But the low data representativeness let us suppose that these results are not valid all over Germany.

In the following a station in the south-western part of Germany is considered. Figure 5 shows the PDF of the monthly precipitation time series observed in Eisenbach-Bubenbach (47.97° N, 8.3° E) in August 1901 and August 2000. In this example, observed changes in August are more pronounced than in Eppenrod. Relatively high and relatively low precipitation has become more unlikely during the observation period. The comparison of the temporal development in the probability of exceedance again for the months January, March, July, August, and November, shown in Fig. 6, reveal more explicitly than in Eppenrod a shift of the yearly maximum probability. While at the beginning of the observation period the maximum probability lays in summer, near the end of the observation the probability of exceedance is highest in autumn and winter. In Fig. 7 the development in the probability of occurrence of relatively low precipitation is shown.

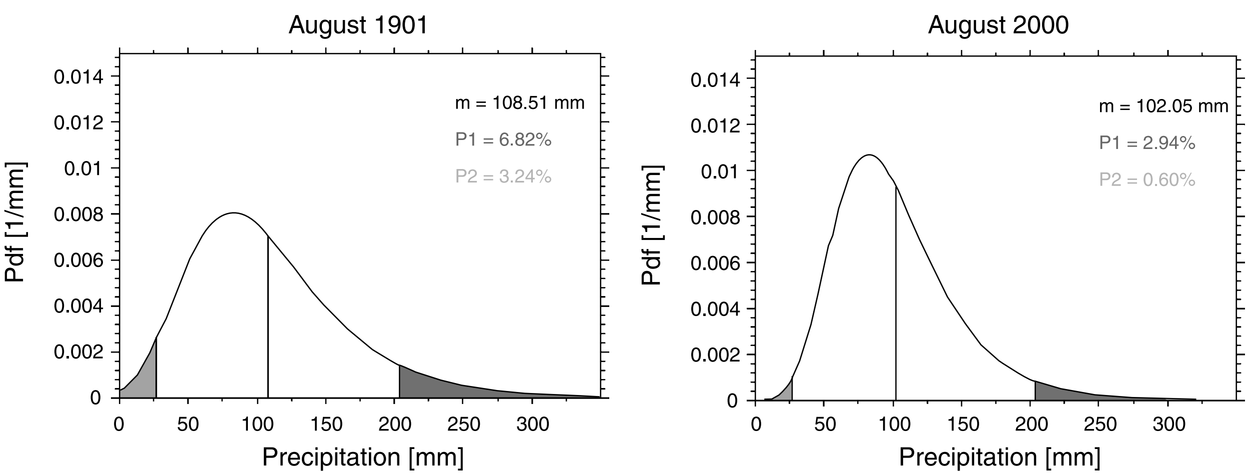

Fig. 5. Same as in Fig. 2 but for Eisenbach-Bubenbach (47.97° N, 8.3° E), Germany, in August

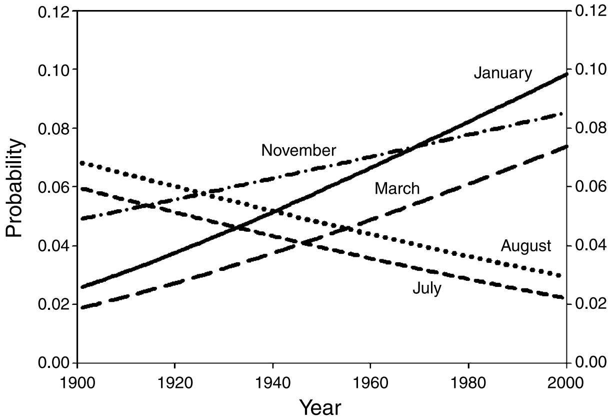

Fig. 6. Same as in Fig. 3 but for the Station Eisenbach-Bubenbach

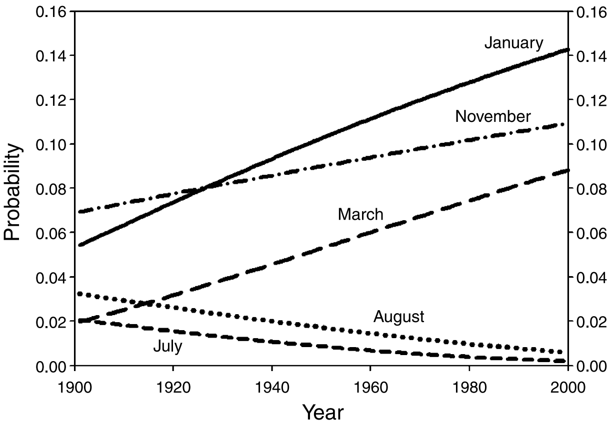

Fig. 7. Same as in Fig. 4 but for the Station Eisenbach-Bubenbach

Some months show decreases and others show increases. As several other stations in Germany, a widening of the distribution is observed in winter. Extreme high and extreme low precipitations are getting more likely during the 20th century. An opposite example can be found in the eastern part of Germany. Figure 8 shows a nearly

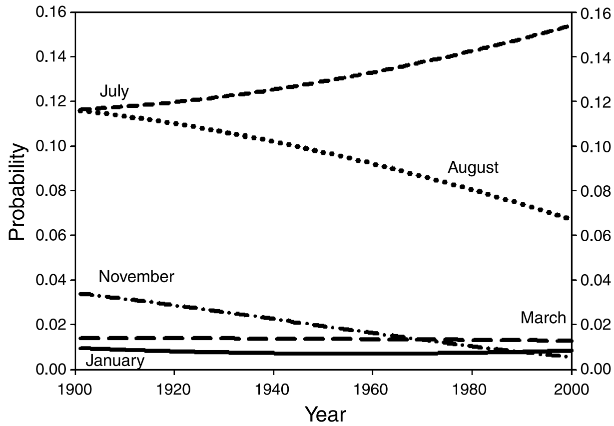

Fig. 8. Same as in Fig. 3 but for the Station Görlitz (51.17° N, 14.9° E)

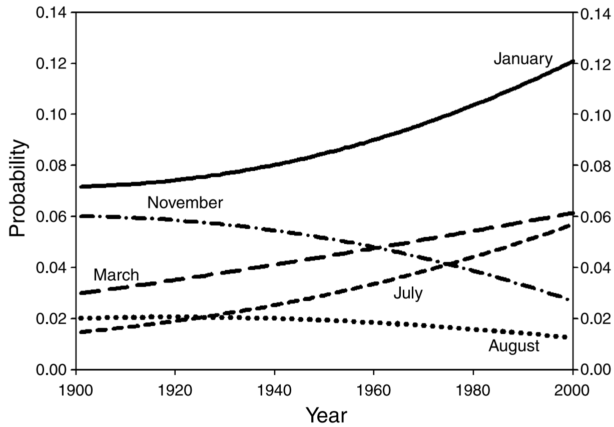

Fig. 9. Same as in Fig. 4 but for the Station Görlitz

constant probability of exceedance in January at the station Görlitz (51.17° N, 14.95° E) and we just observe minor changes in November and March, too. But in summer, tendencies in the occurrence of high precipitation sums in July and August show higher magnitudes but are opposite. Figure 9 shows a pronounced increase

Table 1. Number of stations showing positive (H^+), negative (H^-) and no changes (H°) in the probability of exceeding the 95th percentile. The values A^+ and A^- are the maximum magnitudes of positive and negative changes in the probability and μ_A is the mean value of all magnitudes

	Month											
	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
H°	16	15	15	15	15	16	16	15	15	15	15	15
H^+	110	107	107	100	90	77	75	63	64	79	67	106
H^-	6	10	10	17	27	39	41	54	3	38	50	11
A^+[%]	15.86	16.18	15.20	12.56	9.90	12.32	13.25	12.77	11.15	9.31	12.47	14.83
A^-[%]	−0.94	−1.23	−0.93	−1.42	−2.86	−4.98	−5.62	−8.40	−7.33	−7.62	−5.62	−0.78
μ_A[%]	2.23	1.91	1.87	1.68	1.95	1.84	1.99	0.68	0.78	1.38	1.05	2.36

Table 2. Analogous to Table 1 but for the probability of falling under the 5th percentile

	Month											
	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
H°	15	15	15	15	15	17	15	15	15	16	15	15
H^+	84	74	72	69	54	43	47	33	27	29	24	64
H^-	33	43	45	48	63	72	70	84	90	87	93	53
A^+[%]	10.12	11.22	8.61	6.32	5.25	5.91	4.54	2.71	3.58	6.41	5.98	7.82
A^-[%]	−10.48	−9.17	−6.02	−4.90	−5.28	−3.72	−4.02	−5.43	−5.97	−4.67	−7.65	−12.82
μ_A[%]	0.86	0.90	1.04	0.49	−0.11	−0.24	−0.28	−0.96	−1.09	−0.93	−1.51	−0.14

in the probability of falling under the 5^{th} percentile in January. In November and in August the variability in Görlitz decreases.

Table 1 gives an overview of the results of the entire station network. For every month, the number of stations showing no changes (H°), positive (H^+) or negative (H^-) changes in the probability of exceeding the 95th percentile during the 20th century are given. A positive (negative) change means a higher (lower) probability of exceedance in the considered month of the year 2000 compared to the same month in the year 1901. Changes in the probability of exceedance less than 0.005% are counted as ''no changes''. But even those are significant, because variations in the probability of exceedance deduced from the PDF at different time steps reflect significant detected structures in the estimated location and scale parameter (see Appendix). Additionally, the corresponding maximum magnitudes of positive (A^+) and negative (A^-) changes in the probability of exceedance as well as the mean value (μ_A) of all detected changes in the observation period are given. It can be seen that over the entire year the number of stations with positive changes is larger than the number of stations with negative ones. Especially in winter and spring the occurrence of relatively high precipitation sums increased at the overwhelming majority of stations. In January for example we detect an increase in that probability at 80% of the stations concerned. Most negative changes are detected in summer. Worth mentioning that the maximum magnitudes A^+ between August to April are detected at the station Zugspitze (47.42° N, 10.99° E). This station reveals unusual high magnitudes. Analogous to Table 1, we find in Table 2 the number of stations showing no changes, positive and negative changes in the probability of falling under the 5th percentile during the 20th century. Resuming the information of Tables 1 and 2 we can say that in winter and spring a major part of stations reveal an increase in the occurrence of both kinds of extremes. This means, relatively high and relatively low precipitation become more likely. The opposite effect holds for summer and autumn. In these months negative tendencies prevail again compared to positive ones. The maximum magnitudes are encountered in winter months. This is the case for the positive magnitudes and the negative magnitudes as well. Concerning the probability of exceeding the 95th percentile (Table 1) the mean value μ_A of all detected changes is positive signed in all months. But with regard to the probability of falling under the 5th percentile (Table 2), the mean value μ_A shows a decrease, except from January to April.

In a further step the spatial distribution of detected changes in the probability of extremes is addressed. As selected examples, results of one month in spring, summer, autumn and winter, respectively, are presented. Looking only at the difference in the probability for exceeding the 95th percentile in a specific month in the year 2000 and 1901, the results of a German station network of 132 time series can be provided in a map. The left map in Fig. 10 shows the changes in the probability of occurrence of extreme high precipitation. In August, especially in the northeastern part, decreases are detected. But in the south, we see several increases in the probability of occurrence for high precipitation as well. The results for relatively low precipitation (right map) are more uniform. For the overwhelming majority of stations, we see a decrease in the probability of occurrence. So we come to the conclusion that in summer in the north-eastern part of Germany the extreme high and extreme low precipitation totals are getting more unlikely during the 20th century. But in the south we observe at several stations a shift of the distribution to higher

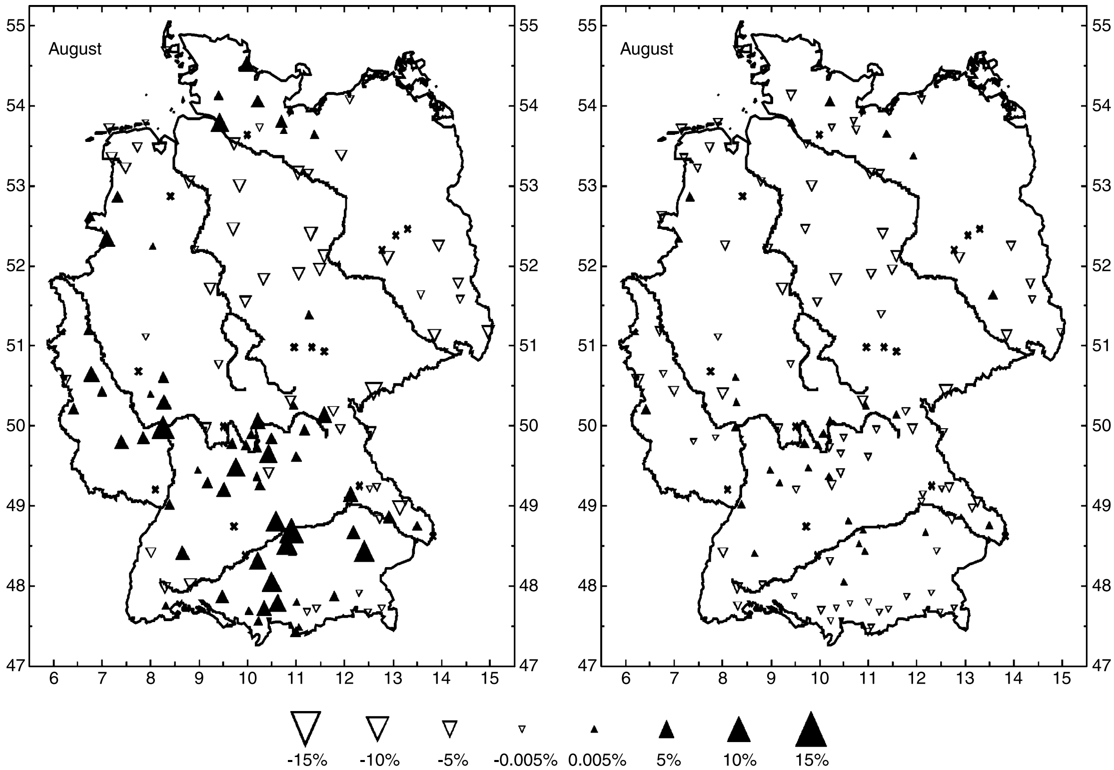

Fig. 10. Changes in the probability of occurrence of a monthly total precipitation larger than the 95th percentile (left) and in the probability of occurrence of a monthly total precipitation smaller than the 5th percentile (right). Results are given for August in the observation period 1901–2000. Upward showing triangles indicate an increase and downward showing triangles indicate a decrease in the probability. The size of the triangle is proportional to the magnitude. Crosses flag up stations, where no changes (<0.005%) are detected in the 20th century

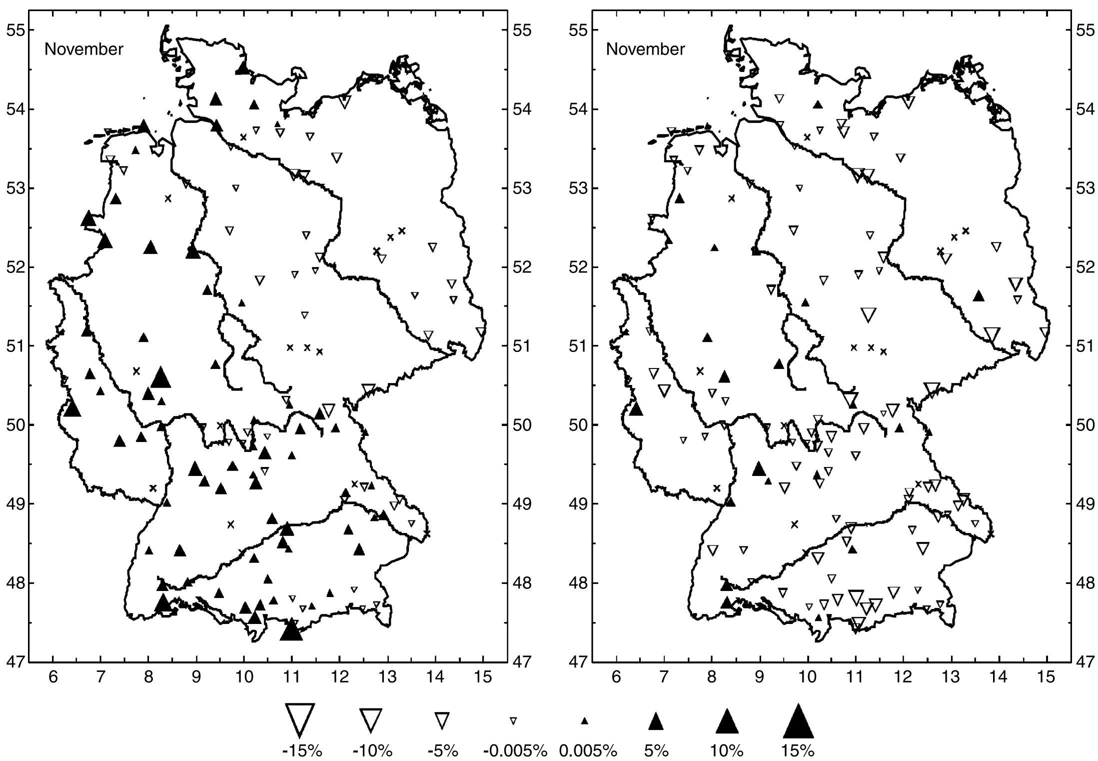

Fig. 11. Analogous to Fig. 10 but for November

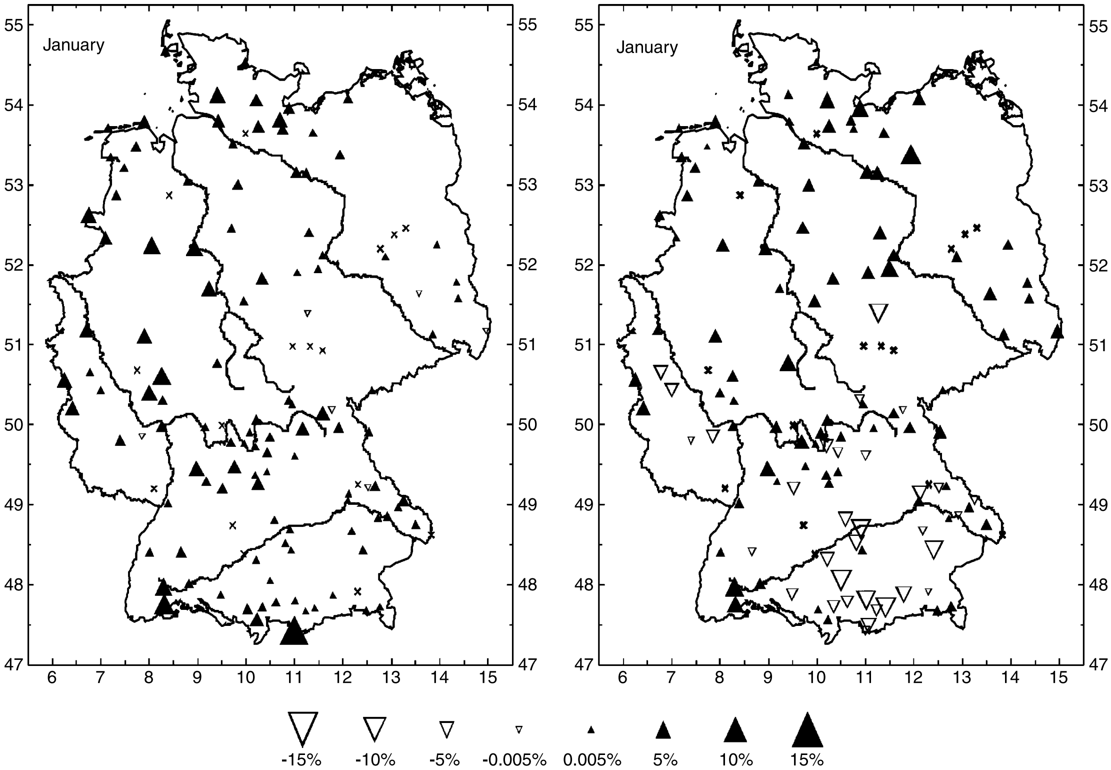

Fig. 12. Analogous to Fig. 10 but for January

values. In Fig. 11 changes in the probability of occurrence of extreme values in November are presented. Compared to Fig. 10 no essential differences concerning the spatial distribution are found. But with regard to the probability of exceeding the 95th percentile the magnitudes, especially in the eastern and southern part of Germany, are smaller in autumn. On the contrary, the overwhelming negative tendencies in the probability of falling under the 5th percentile are more pronounced in autumn than in summer.

Figure 12 shows the changes in the probability of occurrence of extreme values observed in January. The result is an increase in the probability of exceedance in the overwhelming majority of stations. The larger magnitudes can be seen in the western part of Germany. In the east several stations reveal unchanged probabilities or very small changes in the 20th century. But it is not the same for the occurrence of relatively low precipitation. We see significant increases in the northern part of Germany. Even in the east the changes are evident. In the south several stations with decreases can be found. Consequently, the northern part of Germany shows opposite results in summer and winter, respectively. In January, the distribution is widening. Relatively high and relatively low precipitation is getting more likely during the observation period. But it is not the case in the south. In winter and summer as well, the distribution is shifting to higher values at several stations in the south of Germany. Comparable to changes observed between summer and autumn, differences found between winter and spring (see Fig. 13) are again limited to changes in the magnitudes in most cases. The spatial distribution remains the same. In particular, negative tendencies in the probability for falling under the 5th percentile are more pronounced in January than in March.

May be that exclusive minor differences between detected changes in the probability of extremes in summer and autumn on the one hand and changes in winter and spring on the other hand could appear strange to the reader. However, analytical functions describing seasonal variations, trends and low-frequency variations can be detected in the location and the scale parameter of the Gumbel distribution to describe observed changes in the precipitation time series concerned. So, the assumption that climate underlies smooth, long-range changes is implied. Furthermore, the dis-

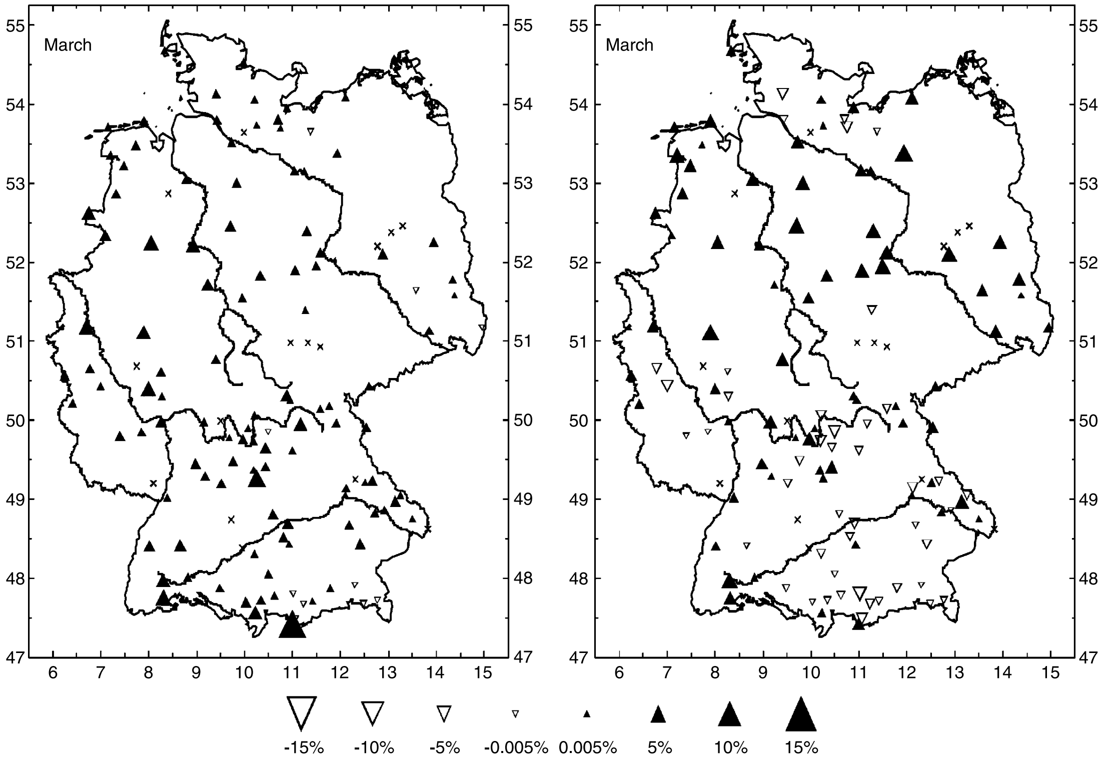

Fig. 13. Analogous to Fig. 10 but for March

tance function used to fit significant functions takes into account the skewness and the pronounced tail of the Gumbel distribution. Consequently, single relatively high values get less weight than in case that the least-squares estimator is used. In other words, the method applied is more robust.

3. Conclusions and outlook

The time series decomposition in a deterministic and a statistical part provides a statistical modelling of monthly precipitation time series. The deterministic part contains the annual cycle and its changes concerning the amplitude and phase shifts, trends up to the order 5 and low frequency variations. Within the analysis of a German station network of 132 precipitation time series these structured components are detected in the location and the scale parameter of the Gumbel distribution, which describes the statistical part of the series. Indeed, the interpretation of the series as a realization of a Gumbel distributed random variable with time-dependent location parameter and time-dependent scale parameter was successful. The application of this method provides a PDF for every time step of the observation period. So, probability assessments of extreme values for every time step of the observation period can be performed.

The results presented in this paper show that tendencies in the probability of occurrence for extreme values of observational monthly precipitation time series in Germany depend on the season and vary from region to region as well. Even though precipitation shows a low data representativeness, some spatial structures can be found in Germany. Most pronounced are the changes in winter. Long-range tendencies in winter (detailed results for January are given) show a widespread increase in the probability of exceeding the 95th percentile, but in the eastern part of Germany, these changes are only small. At some stations the probability of exceedance remains constant during the 20th century or show small decreases. In the western part of Germany pronounced increases go along with a simultaneous increase in the probability for falling under the 5th percentile. So, climate becomes more extreme in this region. On the contrary, in summer and autumn a simultaneous decrease in the probability of occurrence of relatively high and relatively low precipitation can be found in the north-eastern part of Germany.

At some stations where opposite tendencies in winter and summer occur or at stations with pronounced changes in winter, a shift of the maximum probability of exceedance from summer to winter can be observed. In the south, an increase in the probability of occurrence of relatively high and a decrease in the probability of relatively low precipitation are detected at several stations in most months of the year.

The Third Assessment Report (IPCC, 2001) states a 2 to 4% statistically significant increase in the number of heavy precipitation events when averaged across the mid and high latitudes. Easterling et al. (2000), also quoted in this report, compared linear trends of heavy precipitation and total precipitation during the rainy season over several regions of the globe. The magnitudes of changes in heavy precipitation frequencies are always higher than the changes in the mean value. And Mearns et al. (1984) found many years ago that any change in mean goes along with a more intense increase in extremes of one or both signs. Consequently, results of trend analysis from a German station network (Rapp and Schönwiese, 1996) also stand in line with probability assessments given in this study. They found a widespread increase in precipitation in winter and smaller decreases or unchanged mean values in summer. Spatial different trends in the western and eastern part of Germany are retrieved in the probability assessments of extreme values, too.

The analysis of observational German precipitation data was successful on the basis of the Gumbel model. However, in the European scale the interpretation of observed precipitation time series as a realization of a Weibull distributed random variable has to be included because seasonal variations and long-term changes in the shape of the distribution are encountered. The results of a European station network will be subject of a further study.

If the interpretation of precipitation time series as a Gumbel distributed or a Weibull distributed random variable was successful, the provided complete analytical description can be used to calculate the expected value for every time step either. So, statistical modelling represents an alternative approach for estimating broadly used trends in observational precipitation time series. On that way non-Gaussian characteristics can be taken into account and robust estimates can be provided. Additionally, changes in different parameters can be considered and the mean squared error of the trend estimator is smaller using the statistical modelling. Related results will be presented in a further paper.

Acknowledgement

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Appendix

The generalized decomposition technique

On the basis of the least-squares estimator, a successful statistical modelling can only be achieved for temperature time series. The assumption of the least-squares estimator are Gaussian distributed residuals with constant variance. Indeed, only a shift of the Gaussian distribution with constant variance to higher or smaller values in the course of time is necessary to describe observed temperature time series. In order to achieve a complete description of non-Gaussian climate time series structured components like trends, annual cycle and an episodic component is detected in two instead of one parameter of a probability density function (PDF) within the generalized time series decomposition technique. The underlying PDF can be chosen without any further restriction.

The equation

$$S_{j,k}(t) = d_{j\cdot k}t^k \cos\left(2\pi \frac{j}{12} t\right) + e_{j,k}t^k \sin\left(2\pi \frac{j}{12} t\right) \quad (1)$$

with wave number $j = 1, \ldots, 6$ per year and $k = 0, 1, 2$ gives the basis functions to describe the seasonal component. Besides fixed annual cycles, changes in amplitude and phase are allowed. For the amplitude linear and quadratic time dependency is considered. Superposition of several functions $S_{j,k}$ in one time series makes the detection of linear, progressively and degressively shaped changes in phase and amplitude of the annual cycle possible.

In addition, we offer trends up to the order 5 for detections of linear, progressive and degressive trends:

$$T_i(t) = a_i + b_i t^i \quad \text{with} \quad \mathrm{i} = 1, \ldots, 5. \quad (2)$$

Sometimes relatively low-frequency variations can be found superposed on the components mentioned above. Polynomial equations up to the order 5 are used to describe them:

$$P_l(t) = a_o + \sum_{i=1}^{l} a_i t^i. \quad (3)$$

At the end of the time series decomposition procedure, a priori assumed residual distribution is tested on the basis of the Kolmogorov-Smirnov-statistic (Press et al., 1992). In case of a the precipitation analysis presented in the paper, the residuals should be undistinguishable from the realization of a Gumbel distributed random variable G(0,1) with the location parameter 0 and the scale parameter 1 after elimination of significant detected structured components in the parameters. Additional stationarity of the distribution points to a complete description of the time series within the PDF and its time-dependent parameters. The sampling time is divided into two subintervals to compare the corresponding empirical distributions. Again a Kolmogorov-Smirnov test statistic is used. If the statistical tests confirm the assumptions, the time series decomposition was successful and one possible analytical description of the time series is achieved.

The basis of any times series decomposition technique is the distance function and a model selection criterion. An iterative procedure is used to find within these functions the best model equations of two instead of one parameter of a probability density function. Thereby, the stepwise regression (Storch and Zwiers, 1999) is modified for the estimation of two time-dependent parameters of any PDF. The common F-test statistic broadly used in regression analysis to decide whether a specific regressor contributed significantly to explained variance is sensitive to departures from the Gaussian distribution and, therefore, replaced by a test statistic based on the likelihood ratio test (Schrader and Hettmansperger, 1980). It is also worth mentioning that the model equation of one parameter influences the equation for the second parameter and vice versa.

Consistently with the maximum likelihood principle another distance function (Huber, 1981), defined as the negative logarithm of the PDF, replaces the function of squared errors to be minimized. With the exchange of the distance function, fitted basis functions describe changes in the location, scale or shape parameter of an appropriate PDF. As an example, the PDF of the Gumbel distribution, assuming time-dependent parameters, is

$$f(x,t) = \frac{1}{b(t)} \left\{ \exp\left(-\frac{x-a(t)}{b(t)} \right) \exp[-e^{-(x-a(t))/b(t)}] \right\}, \tag{4}$$

where $a(t)$ denotes the location and $b(t)$ the scale parameter. The corresponding distance function is

$$\rho(x,t) = \ln(b(t)) + \exp\left(-\frac{x-a(t)}{b(t)} \right) + \frac{x-a(t)}{b(t)}. \tag{5}$$

The alternative distance function takes the non-Gaussian characteristics into account. For normally distributed errors, the influence increases quadratically with the distance of the points from the mean value. The more prominent tail of the asymmetric Gumbel distribution provides an asymmetric distance function showing a smaller increase with the distance from the location parameter.

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