

Übungsblatt 9

READING ASSIGNMENT

The goal of this assignment is to become acquainted with *discriminants* and *resultants*, together with the main statements about them. The literature is

[Bo]: Bosch “Algebra” (Springer, 9th edition, 2020), eBook [here](#).

Program. Read the beginning of [Bo, Chapter 4.4] up to the statement of the Fundamental Theorem on symmetric polynomials (included). Then pass directly to the second application of such theorem, i.e. the discussion about the discriminant of a monic polynomial; see [Bo, second paragraph of p. 224]. Continue reading until the end of the examples on Δ_f given in [Bo, p. 232]. Pay particular attention to the final three corollaries.

Aufgabe 1 (4 Punkte)

For a ring R , verify the following formulas for discriminants Δ_f of polynomials $f \in R[X]$.

(i) If $f = X^2 + aX + b$, then $\Delta_f = a^2 - 4b$.

(ii) If $f = X^m + aX + b$, where $m \geq 2$, then

$$\Delta_f = (-1)^{m(m-1)/2} ((1-m)^{m-1} a^m + m^m b^{m-1}).$$

(iii) If $f = X^3 + aX^2 + bX + c$, then $\Delta_f = a^2b^2 + 18abc - 4a^3c - 4b^3 - 27c^2$.

(iv) If $f = X^4 + aX^2 + bX + c$, then

$$\Delta_f = 144ab^2c - 128a^2c^2 - 4a^3b^2 + 16a^4c - 27b^4 + 256c^3.$$

Aufgabe 2 (4 Punkte)

Compute all *rational* solutions of the following system of equations.

$$\begin{cases} -3X^4Y + 2X^3Y^2 - 6X^4 + 20X^3Y - 6X^2Y^2 + 32X^3 - 42X^2Y + 6XY^2 - \\ \quad - 59X^2 + 35XY - 2Y^2 + 44X - 9Y - 9 = 0 \\ X^3Y^2 + X^2Y^3 + 4X^3Y + 3X^2Y^2 - 2XY^3 + 4X^3 - 9XY^2 + Y^3 - 4X^2 - \\ \quad - 14XY + 6Y^2 - 7X + 14Y + 11 = 0 \end{cases}$$

Hint: the computation of resultants of high degree polynomials is too painful to be made by hand. Try to use a computer algebra system! Suggestion: SageMath. It is based on Python, is free, and can be used directly online e.g. on CoCalc. You can compute resultants and roots of polynomials in one variable, with direct commands.

Please, upload your solutions on the [Olat page](#) of this course, by **14:00** on **Tuesday, 19.01.2021**.