Goethe-Universität Frankfurt
Institut für Mathematik
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Algebra
Prof. Dr. Martin Möller
M.Sc. Jeonghoon So
M.Sc. Riccardo Zuffetti

Übungsblatt 6

Aufgabe 1 (4 Punkte)

Let K be a field of characteristic $\neq 2$ and $f \in K[X]$ a seperable irreducible polynomial with zeros $\alpha_1, ..., \alpha_n$ in a splitting field L of f over K. Assume that the Galois group of f is cyclic of even order and show:

(a) The discriminant $\Delta = \prod_{i<j}(\alpha_i - \alpha_j)^2$ does not admit a square root in K.

(b) There is a unique intermediate field E of L/K satisfying $[E : K] = 2$, namely $E = K(\sqrt{\Delta})$.

Aufgabe 2 (4 Punkte)

Determine the Galois groups of the following polynomials in $\mathbb{Q}[X]$:

(a) $X^3 + 6X^2 + 11X + 7$

(b) $X^3 + 3X^2 - 1$

(c) $X^4 - 4X^2 - 6$

Aufgabe 3 (4 Punkte)

Write down

$$f = X^3Y^3 + X^3Z^3 + 7X^2Y^2Z^2 + Y^3Z^3 \in \mathbb{Q}[X, Y, Z]$$

as a $\mathbb{Q}$-linear combination of products of elementary symmetric polynomials.
Hint: Proof of Satz 5 from chapter 4.3 (Bosch, Algebra).

Aufgabe 4 (4 Punkte)

(a) Let $n \in \mathbb{N}$ with $n \geq 3$ and ζ a primitive n-th root of unity (primitive n-te Einheitswurzel). Show that

$$[\mathbb{Q}(\zeta + \zeta^{-1}) : \mathbb{Q}] = \frac{\varphi(n)}{2}.$$

Here φ denotes Euler's phi function.

(b) Let K_8 be the 8th cyclotomic field (Kreisteilungskörper), so the splitting field of $X^8 - 1$ over $\mathbb{Q}$. Compute the Galois group and all intermediate fields of $K_8/\mathbb{Q}$.

Please, upload your solutions on the Olat page of this course, by **14:00** on **Tuesday, 15.12.2020**.