

Übungsblatt 10

Aufgabe 1 (2 Punkte)

Let G be a group, $P \subset G$ a p -Sylow subgroup and $N \subset G$ a normal subgroup. Show that $P \cap N$ is a p -Sylow subgroup of N .

Aufgabe 2 (6 Punkte)

- (a) Let $H \subset G$ be a normal subgroup. Show that G is solvable if and only if G/H and H are solvable.
- (b) Show that every group of order < 60 is solvable. Why is this false for $= 60$?

Aufgabe 3 (4 Punkte)

Let K be a field and

$$T_n(K) = \{A = (a_{ij}) : a_{ij} = 0 \text{ for } i > j, a_{ii} \neq 0\} \subset \text{GL}_n(K)$$

be the group of invertible upper-triangular matrices. Let

$$U_n(K) = \{A = (a_{ij}) : a_{ij} = 0 \text{ for } i > j, a_{ii} = 1\} \subset T_n(K)$$

be the normal subgroup of unitriangular matrices. Let

$$\text{Diag}_n(K) = \{A = (a_{ij}) : a_{ij} = 0 \text{ for } i \neq j, a_{ii} \neq 0\} \subset \text{GL}_n(K)$$

be the abelian subgroup of diagonal matrices.

- (a) Find a surjective group homomorphism

$$\pi_n : T_n(K) \rightarrow \text{Diag}_n(K).$$

- (b) Find a surjective group homomorphism

$$U_n(K) \rightarrow U_{n-1}(K).$$

- (c) Prove that $T_n(K)$ is solvable.

Aufgabe 4 (4 Punkte)

- (a) Write the group D_n of symmetries of the regular n -polygon as the semidirect product of two cyclic groups. Show that D_n is solvable.
- (b) Write the group $T_n(K)$ as the semidirect product of $U_n(K)$ and some other group.