Goethe-Universität Frankfurt
Institut für Mathematik
Wintersemester 2018
4. Februar 2019

Komplexe Algebraische Geometrie
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Übungsblatt 13

Aufgabe 1 (3 Punkte)

Show that any complex manifold admits an hermitian structure.
Hint: from Differential Geometry we know that *every* differential manifold (in particular *every* complex manifold) admits a Riemannian metric.
Let g be a Riemannian metric on a complex manifold X with induced almost complex structure I. Use g and I to define an hermitian structure h.

Aufgabe 2 (3 Punkte)

Let X and Y be two Kähler manifolds. Show that the product $X \times Y$ is Kähler.
Hint: using the pull-back, define on $X \times Y$ the (integrable) almost complex structure, the hermitian structure and the Kähler form from the ones on X and Y.

Aufgabe 3 (7 Punkte)

(a) Show that the Fubini–Study Kähler form ω_{FS} on $\mathbb{P}^n$ is positive definite, i.e. that ω_{FS} really is the Kähler form associated to a metric.
Hint: verify this on each standard open U_j separately.

(b) Prove that

$$\int_{\mathbb{P}^1} \omega_{\mathrm{FS}} = 1.$$

Hint: compute the integral locally, i.e. on $\mathbb{C}$. Change coordinates from $\mathbb{C}$ to $\mathbb{R}^2$, then compute the integral in polar coordinates.

Aufgabe 4 (3 Punkte)

Let $\mathbb{C}^n \hookrightarrow \mathbb{C}^{n+1}$ be the standard inclusion $(z_0, \ldots, z_{n-1}) \mapsto (z_0, \ldots, z_{n-1}, 0)$ and consider the induced inclusion $\mathbb{P}^{n-1} \hookrightarrow \mathbb{P}^n$. Show that restricting the Fubini–Study Kähler form $\omega_{\mathrm{FS}} \in \mathcal{A}^{1,1}(\mathbb{P}^n)$ from $\mathbb{P}^n$ to $\mathbb{P}^{n-1}$ yields the Fubini–Study Kähler form on $\mathbb{P}^{n-1}$.

Abgabe Zu Beginn der Vorlesung um **14:00** am **Montag, den 11. Februar**.